Nonlinear Dispersive Equations

Conference in honor of Jean-Claude Saut on the occasion of his 75^{th} birthday

Institut de Mathématiques de Toulouse
May 17-19, 2022

Tuesday, May 17

14:00 - 15:00 Patrick GÉRARD

On a derivative nonlinear Schrödinger equation on the Hardy space of the line

We introduce a nonlinear Schroedinger equation on the line, with a mass critical non-local cubic nonlinearity of DNLS type, which conserves the Hardy property of a Fourier transform supported in the positive half line. We identity a Lax pair for this equation, and we use this structure for studying multisoliton solutions. This a jointwork with Enno Lenzmann (Basel).

15:00 - 16:00 Philippe GRAVEJAT

On the stability of the Ginzburg-Landau vortex

We present a recent work in collaboration with Eliot Pacherie (New York University at Abu Dhabi) and Didier Smets (Sorbonne University) about the vortex solution (with degree one) of the Ginzburg-Landau equation. We first establish that it minimizes a renormalized version of the Ginzburg-Landau energy. We then deduce that it is orbitally stable along the Hamiltonian flow given by the Gross-Pitaevskii equation.

16:30 - 17:30 Eliot PACHERIE

Uniqueness of the two vortex travelling wave in the Gross-Pitaevskii equation

For the Gross-Pitaevskii equation in dimension 2, it has been shown that there exists a minimizer of the energy at fixed momentum, and that any such minimizer is a travelling wave, behaving like two well separated vortices. In this talk we will discuss the uniqueness of this minimizer, as well as its orbital stability. This is a joint work with David Chiron.

17:30 - 18:30 Simona ROTA NODARI

On a nonlinear Schrödinger equation: uniqueness, non-degeneracy and applications

I will first state a general result about the uniqueness and the non-degeneracy of positive radial solutions to some semi-linear elliptic equations $-\Delta u = g(u)$. Then I will consider the case of the double power non-linearity $g(u) = u^q - u^p - \mu u$ for $p > q > 1$ and $\mu > 0$. In this case, the non-degeneracy of the unique solution u_μ allows us to derive its behavior in certain regimes of the parameter μ. This implies the uniqueness of energy minimizers at fixed mass and gives some important information about the orbital stability of u_μ. Joint work with Mathieu Lewin.

Wednesday, May 18

9:00 - 10:00 Evelyne MIOT

Dynamics of several point vortices for the lake equations

We study the asymptotic dynamics of point vortices for the lake equations with positive depth, when the vorticity is initially sharply concentrated around N points. More precisely, we show that the vorticity remains concentrated - weakly in all directions and strongly in the direction of the steepest ascent of the depth function - around N points for all times, and that the trajectories follow the level lines of the depth function. This is joint work with Lars Eric Hientzsch (Universität Bielefeld) and Christophe Lacave (Université Grenoble Alpes)

10:00 - 11:00 Nikolay TZVETKOV

Universality results for a class of nonlinear wave equations

11:30 - 12:30 Nicola VISCIGLIA

The Nonlinear Schrödinger Equation with space white-noise potential and general nonlinearity

We show global well posedness of NLS with nonlinearity arbitrarily large and with multiplicative white-noise potential.

In case the nonlinearity is not too big we rely on suitable energy estimates; in case of large nonlinearities we need to exploit more deeply the dispersion leading to suitable Strichartz estimate with a logarithmic loss. This loss can be absorbed by using probabilistic arguments. The talk is based on joint works with N. Tzvetkov.

14:00 - 15:00 Jerry L. BONA

Complex-valued solutions of the Korteweg-de Vries equation

We will discuss certain classes of complex-valued solutions of the classical Korteweg-de Vries equation. Included are results of existence, blow up and non-existence. This is joint work with Fred Weissler.

15:00 - 16:00 Felipe LINARES

Local energy decay for solutions of the Benjamin-Ono equation

In this lecture I will present recent results regarding the asymptotic behavior of solutions to the initial value problem associated with the Benjamin-Ono equation. We use new techniques in order to show that solutions of this system decay to zero in the energy space in an appropriate domain. The result is independent of the integrability of the equation involved and it does not require any size assumptions. We also consider the asymptotic behavior of the solution in a domain moving in time in the right direction. Finally, we discuss the decay of the solution in the far left region.

This is a joint work in collaboration with R. Freire (IMPA), C. Muñoz (UChile) and G. Ponce (UCSB).

16:30 - 17:30 Christian KLEIN

Numerical study of Korteweg-de Vries equations

We present a numerical study of solutions to equations from the family of Korteweg-de Vries equations: Korteweg-de Vries, Kadomtsev-Petviashvili and Zakharov-Kuznetsov equations. We study the stability of solitons, dispersive shock waves and potential blow-ups.

17:30 - 18:30 Charlotte PERRIN

Well-posedness of partially congested Navier-Stokes equations

In this talk, I will address the mathematical analysis of 1D Navier-Stokes equations including a maximum packing constraint, that is a maximal constraint on the density. These equations arise naturally in the modeling of mixtures like suspensions or in the modeling of collective motion. The main feature of the model is the co-existence of two different phases. In the congested phase, the pressure is free and the dynamics is incompressible, whereas in the non-congested phase, the fluid obeys a pressureless compressible dynamics. I will discuss the Cauchy problem for initial data which are small perturbations in the non-congested zone of travelling wave profiles. This is a joint work with Anne-Laure Dalibard.

Thursday, May 19

9:00 - 10:00 David LANNES

An obstacle to the long-time existence for Boussinesq System

Boussinesq systems model the propagation of waves in shallow water and in a weakly nonlinear regime. Several versions of these sysems exist, that are formally equivalent at the precision of the model. One of the recent achievements of Jean-Claude and his collaborators was to prove the long time existence for all these systems, in the sense that the existence time is large enough for the dispersive and nonlinear effects to be relevant at leading order. Do these results remain valid if we consider the same phsyical configuration, but with a partially immersed object somewhere ? This raises new difficulties, such as the appearance of dispersive boundary layers that need to be controled.

10:00 - 11:00 Luis VEGA

New Conservation Laws and Energy Cascade for 1d Cubic NLS

I'll present some recent results concerning the IVP of 1d cubic NLS at the critical level of regularity. I'll also exhibit a cascade of energy for the 1D Schrödinger map which is related to NLS through the so called Hasimoto transformation. For higher regularity these two equations are completely integrable systems and therefore no cascade of energy is possible.

11:30 - 12:30 Stefan LE COZ

Ground states on nonlinear quantum graphs

The nonlinear Schrödinger equation is an ubiquitous model in physics, with numerous applications in areas as divers as Bose-Einstein condensation or nonlinear optics. In many physical situations, the underlying space is essentially one dimensional and can be modeled as a metric graph, i.e. a collection of vertices and edges with finite or infinite lengths. The mathematical study of this type of model is very recent and is gaining an incredible momentum. In this talk, we will review some of the results concerning the existence of nonlinear Schrödinger ground states on graphs and present a numerical approach for their computation. This is a joint work with Christophe Besse and Romain Duboscq.

14:00 - 15:00 Didier PILOD

Dispersive perturbations of Burgers and hyperbolic equations

In this talk, we will review several results on dispersive perturbations of Burgers and hyperbolic equations in one dimension. We will also discuss more recent results dealing with 2-dimensional nonlocal nonlinear dispersive equations such as fractional KP equations and the full dispersion KP equations.

This talk is based on several joint works in collaboration with Jean-Claude Saut, Felipe Linares, Christian Klein, Sigmund Selberg and Achenef Tesfahun.

15:00 - 16:00 Gérard IOOSS

Patterns and quasipatterns from the superposition of two hexagonal lattices

We consider the Swift - Hohenberg PDE with quadratic as well as cubic nonlinearities, and look for solutions built with fhe superposition of two hexagonal lattices rotated by an angle β with respect to each other. We prove existence of several new types of quasi-patterns, in particular quasipatterns made from the superposition of hexagons and stripes (rolls) oriented in almost any direction and with any relative translation, and quasipatterns made from the superposition of hexagons with unequal amplitude (provided the coefficient of the quadratic nonlinearity is small). We consider the periodic case as well, and extend the class of known solutions, including the superposition of hexagons and stripes. For the quasiperiodic cases, the proofs follow the process used by the author with B. Braaksma and L. Stolovitch on a simpler problem.

This lecture is based on a joint work with A. M. Rucklidge (Leeds).

https://math.unice.fr/ iooss/publis1/Io-Ruck2020.pdf